How do Performance Indicator Parametrizations Influence the Assessment of Algorithm Portfolios?

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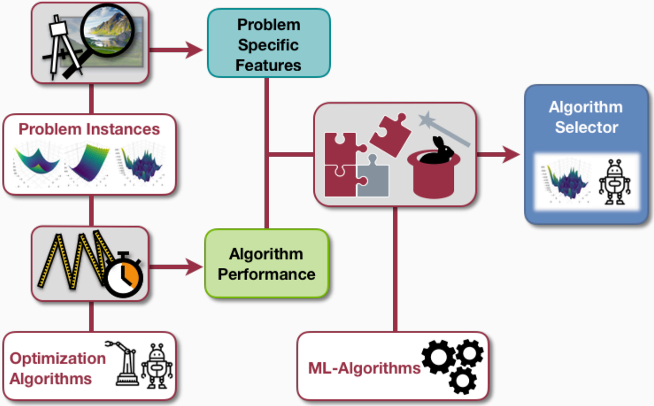
Introduction

Algorithm Selection

Idea of Algorithm Selection:

- *Algorithm Selection Problem*[1]: find the individually best suited algorithm for an unseen optimization problem



1. Rice, J. (1976). *The Algorithm Selection Problem.* In Advances in Computers (pp. 65-118).

Algorithm Selection

Requirements:

- Comprehensive benchmark of portfolio solvers (as a foundation for algorithm selection)
- Suitable performance measure needed, e.g., PAR10[2], ERT[3].
- Performance measures often parameterized.
  $\rightsquigarrow$ How do parameters affect the benchmark results?

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2. Bischl, B. et al. (2016). *ASlib: A Benchmark Library for Algorithm Selection.* In Artificial Intelligence Journal (pp. 41-58).
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Our contribution:

- Systematic analysis of parameterizations on a comprehensive benchmark study of inexact TSP solvers.

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- $\rightsquigarrow$ empirical runtimes $r_1^{A,I}, \ldots, r_m^{A,I}$.

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- $\leadsto$ empirical runtimes $r_1^{A,I}, \dots, r_m^{A,I}$.
- Time limit / cutoff time $T \in \mathbb{R}_{>0}$.

Performance Measures

Numeric Example:

- 10 runs of solvers X and Y
- budget for runtime r_s of successful runs is set to $T = 1$
- solver X: 8 successful runs ($p_f = 0.2$) with $r_s = 0.8$
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$\leadsto$ How do we aggregate the runs (meaningfully)?

Performance Measures (cont.)

Penalized Average Runtime (PAR)[4]:

- Arithmetic mean of running times, $r_i^{A,I}, i \in [m]$
- Expired runs are penalized by $f \cdot T$, where f is the **penalty factor**

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- Usually, the rather heuristic value $f = 10$ is employed.
  $\rightsquigarrow PAR_{A,I}(10)$

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Performance Measures (cont.)

Penalized Quantile Runtime (PQR)[5]:

- Replace outlier-sensitive mean by more robust p-**quantile**, $p \in (0, 1]$.

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$$\text{PQR}_{A,I}(p, f) := \begin{cases} f \cdot T, & \text{if } \sum_{i=1}^{m} \mathbb{1}\{r_i^{A,I} < T\} < \lfloor mp + 1 \rfloor \\ q_p(r_1^{A,I}, \ldots, r_m^{A,I}), & \text{otherwise.} \end{cases}$$

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Expected Runtime (ERT)[6]:

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$$\mathrm{ERT}_{A,I} = \frac{1}{s}\sum_{j=1}^{s} r_{i_j}^{A,I} + \left(\frac{1-p_s}{p_s}\right)\cdot T$$

$$= \frac{1}{s}\left(\sum_{j=1}^{s} r_{i_j}^{A,I} + (m-s)\cdot T\right).$$

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- Corresponds to average runtime for observing one successful run.

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Penalized Expected Runtime (PERT):

- Introducing penalty factor into ERT.

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$$\mathrm{PERT}_{A,I}(f) = \frac{1}{s}\sum_{j=1}^{s} r_{i_j}^{A,I} + \left(\frac{1-p_s}{p_s}\right)\cdot f \cdot T$$
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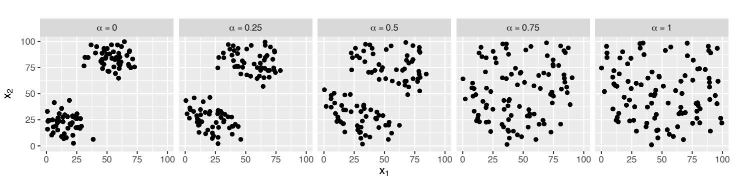
Performance Indicator	$f = 10$		$f = 100$	
	X	Y	X	Y
$PAR_{A,I}(f)$	2.64	8.04	20.64	80.04
$PQR_{A,I}(0.5, f)$	0.80	10.00	0.80	100.00
$ERT_{A,I}$	1.05	4.20	1.05	4.20
$PERT_{A,I}(f)$	3.30	40.20	25.80	400.20

Case Study

Case Study

- Based on performance data from our previous TSP algorithm selection study[7]:
- Five state-of-the-art inexact TSP solvers (**Algorithms** $\mathcal{A}$):
  - MAOS [4], EAX [3], LKH [2], EAX+restart and LKH+restart [1].
- Six sets of TSP instances (**Problems** $\mathcal{I}$):
  - VLSI, TSPLIB, National, RUE, clustered (netgen) and morphed.



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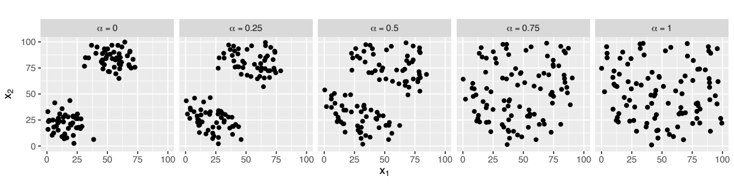

- EAX+restart was single-best-solver (SBS) regarding $PAR_{A,I}(10)$.

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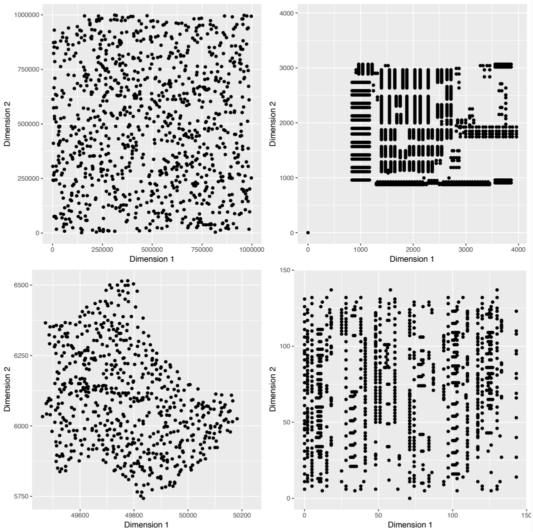

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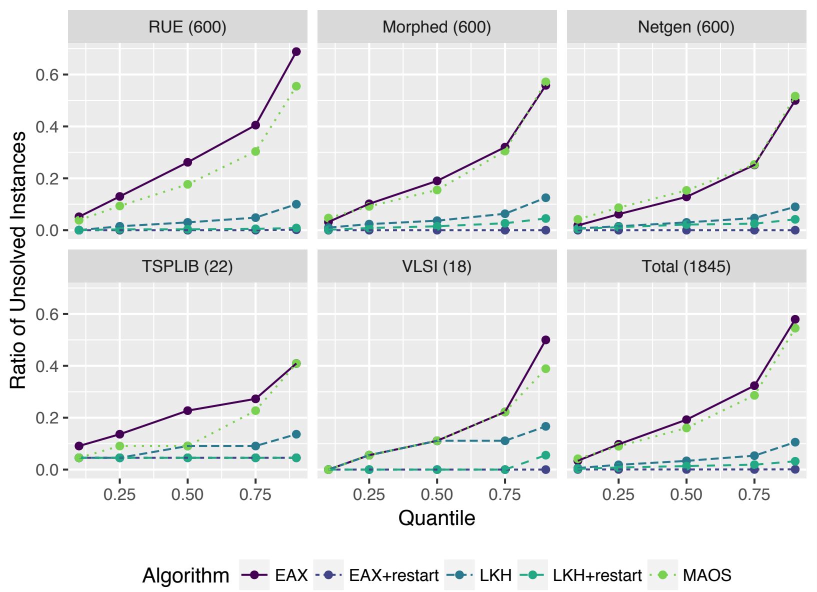
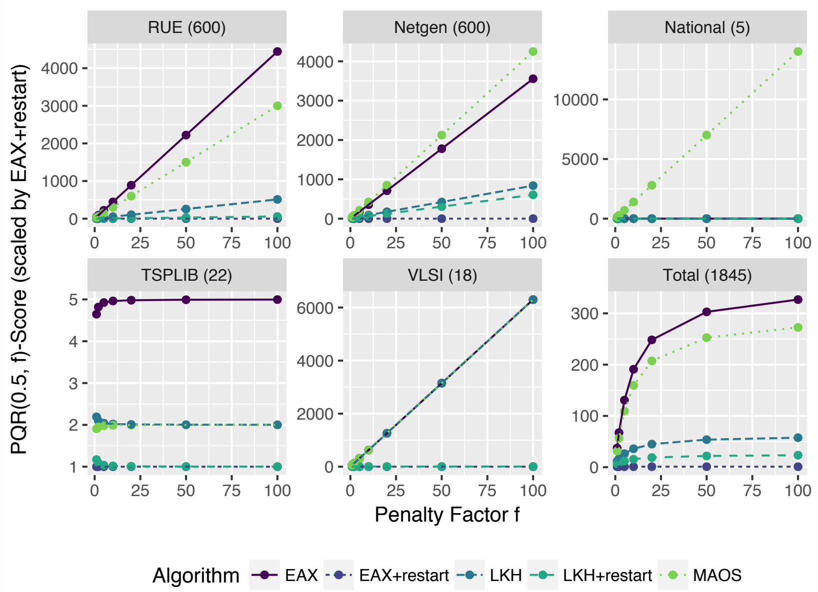
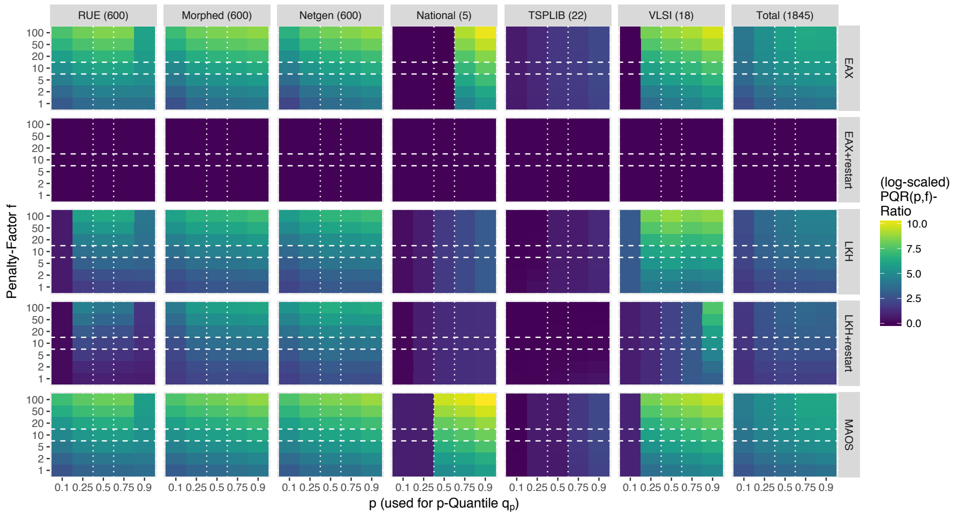


RUE (600)
Morphed (600)
Netgen (600)
TSPLIB (22)
VLSI (18)
Total (1845)
Ratio of Unsolved Instances
0.0
0.2
0.4
0.6
0.25
0.50
0.75
Quantile
Algorithm
EAX
EAX+restart
LKH
LKH+restart
MAOS

Case Study


RUE (600)
Netgen (600)
National (5)
TSPLIB (22)
VLSI (18)
Total (1845)
PQR(0.5, f)-Score (scaled by EAX+restart)
Penalty Factor f
Algorithm
EAX
EAX+restart
LKH
LKH+restart
MAOS

Case Study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EAX
EAX+restart
LKH
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MAOS
Penalty-Factor f
100
50
20
10
5
2
1
0.1 0.25 0.5 0.75 0.9
p (used for p-Quantile q_p)
(log-scaled) PQR(p,f)-Ratio
10.0
7.5
5.0
2.5
0.0

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Observations:

- Finding a suitable pair of penalty factor f and quantile p quickly becomes very complex.
- (Visual) Comparison of solvers also becomes more difficult.

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Idea:

- One basically wants to optimize the runtime <u>and</u> success probability simultaneously.
- Why not use a multi-objective approach?
  $\rightsquigarrow$ consider for instance HV principle

Numeric Example:

- solver X: 8 successful runs ($p_f = 0.2$) with $r_s = 0.8$
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$$\mathrm{HV}_{A,I} = \left(T - r_s\right) \cdot \left(1 - p_f\right).$$

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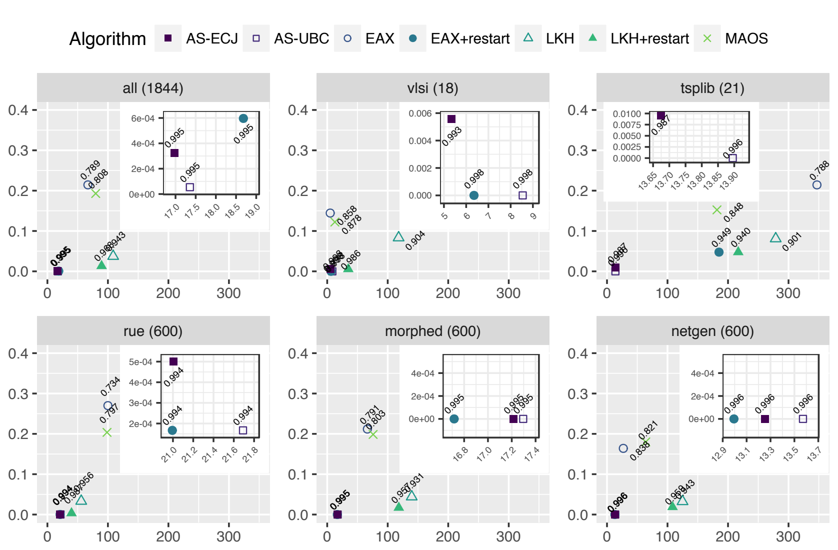
(Visual and Measure Independent) Comparison of TSP Solvers:



Further Comparison of Performance Measure Parameterizations

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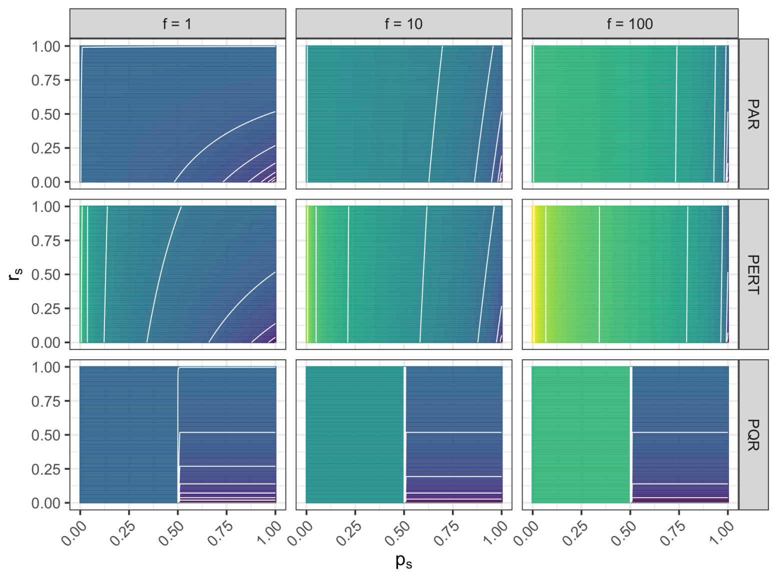
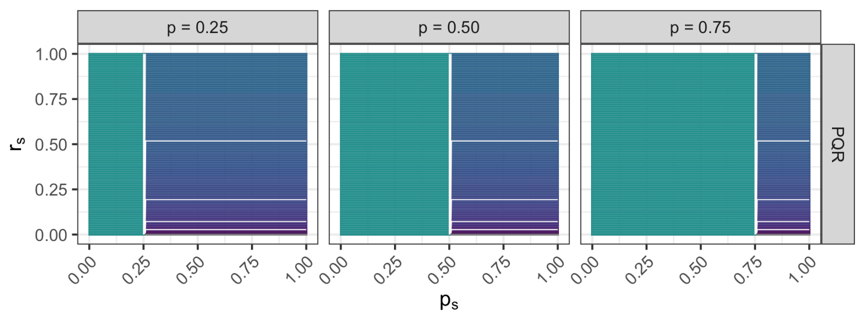
(Theoretical) Effect of Penalty Factor on Performance Measures:



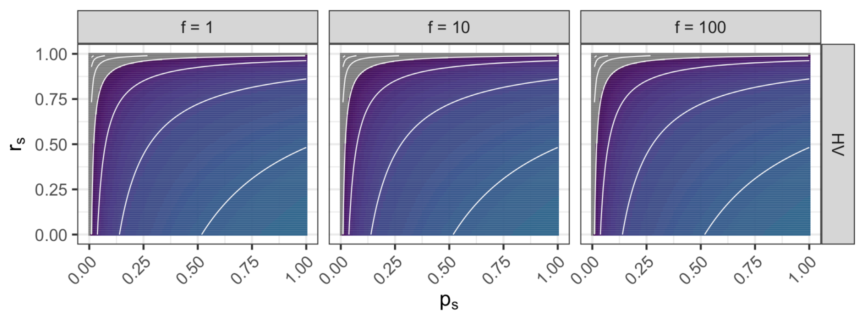
Further Comparison of Performance Measure Parameterizations

(Theoretical) Effect of Quantile on PQR(p, 10)-Score:



Further Comparison of Performance Measure Parameterizations

HV indicator as performance measure:



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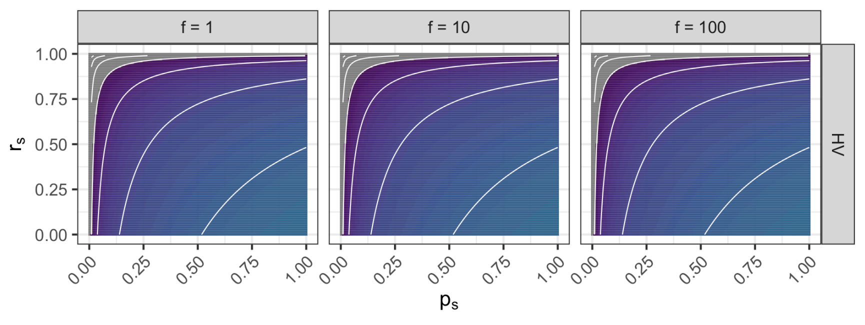

Note that HV is robust against alterations of the penalty score.

Conclusion & Outlook

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Conclusion:

- We systematically analyzed effects of different parameterizations of performance indicators.
- Varying penalty factor allows for altering leverage of failed runs.
- (P)ERT is much more prone to single runs
  $\rightsquigarrow$ huge impact of single failed runs.
- Choosing a suitable measure has a huge impact on the actual performance assessment (for solvers and selectors).
- HV might be a good alternative to common measures.

Outlook:

- Theoretical investigations of indicators.
- Introduction of alternative (multi-objective) indicators[8].
- Application in context of algorithm selection.

8. Bossek, J. & Trautmann, H. (2018). *Multi-Objective Performance Measurement: Alternatives to PAR10 and Expected Running Time.* In Proceedings of LION 2018.

Questions?

References

[1] Jérémie Dubois-Lacoste, Holger H. Hoos, and Thomas Stützle. On the Empirical Scaling Behaviour of State-of-the-art Local Search Algorithms for the Euclidean TSP. In **Proceedings of the 17th Annual Conference on Genetic and Evolutionary Computation (GECCO)**, pages 377 – 384. ACM, 2015. ISBN 978-1-4503-3472-3. doi: 10.1145/2739480.2754747. URL `http://dl.acm.org/citation.cfm?id=2754747`.

[2] Keld Helsgaun. General k-opt Submoves for the Lin-Kernighan TSP Heuristic. **Mathematical Programming Computation**, 1(2-3):119 – 163, 2009. doi: 10.1007/s12532-009-0004-6. URL `https://link.springer.com/article/10.1007/s12532-009-0004-6`.

[3] Yuichi Nagata and Shigenobu Kobayashi. A powerful genetic algorithm using edge assembly crossover for the traveling salesman problem. **INFORMS Journal on Computing**, 25(2):346 – 363, 2013. doi: 10.1287/ijoc.1120.0506. URL `http://dl.acm.org/citation.cfm?id=2466704`.

[4] Xiao-Feng Xie and Jiming Liu. Multiagent Optimization System for Solving the Traveling Salesman Problem (TSP). **IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics**, 39(2):489 – 502, 2009. URL `http://ieeexplore.ieee.org/document/4717264/`.